DEPENDENCE OF COMPTON LINE BREADTH ON PRIMARY WAVE-LENGTH WITH THE MULTI-CRYSTAL SPECTROGRAPH

By Jesse W. M. DuMond and Harry A. Kirkpatrick
California Institute of Technology, Pasadena

(Received August 11, 1931)

Abstract

The present report is a continuation of the work reported previously. The main purpose of the research is to test the correctness of the assumption that the initial velocities of electrons in the scattering body cause the observed breadth of the Compton shifted line. In the previous paper the natural breadths of the Compton line were observed for different angles of scattering and the breadth was found to obey the functional dependence on scattering angle predicted by DuMond on the assumption that electron velocities cause the breadth. According to DuMond's theory the breadth should also be nearly proportional to the primary wave-length. This point is tested in the present paper. Exposures of about one thousand hours each were made with characteristic K radiation from molybdenum, silver and tungsten target tubes. In each case the radiation was scattered from a graphite scatterer at a very well defined large scattering angle of 156°, the multicrystal spectrograph being used to analyze the scattered radiation. Reproductions of the photographic spectrograms are shown and also microphotometer curves of the spectra. The breadth of the Compton line is found to diminish with shorter primary wave-lengths in complete accord with the predictions of DuMond's theory. Unless some other cause can be found to explain the observed behavior of the breadth the results of this paper and the above mentioned previous paper constitute a complete vindication of this theory and of its basic assumption that the initial velocities of the electrons in the scatterer cause the breadth of the Compton shifted line. If this is correct, then these experiments constitute direct experimental evidence of the dynamic nature of atoms. The Compton shifted line can indeed be thought of as broadened by the Doppler effect of the motion of the scattering electrons. A simplified form of the theory of modified scattering by initially moving electrons is presented to supplement and clarify the more elaborate and exact theory of the previous paper. We call attention to the fact that the theory and experimental results are in no way discordant with wave mechanics or the uncertainty principle but we believe that to translate the present exposition into the language of wave mechanics would tend to obscure rather than clarify the picture. The results of this work are so clearly defined that the reality of the much narrower Compton lines obtained recently by several investigators seems, in our opinion, to be very doubtful.

Purpose of the Investigation

In a recently published paper[1] the authors have described a study undertaken to test whether the initial velocities of electrons in a solid body of low atomic number scattering x-rays cause the observed broadening of the Compton shifted line. A theory based on the assumption under test was presented in that paper which predicted the behavior of the breadth of the shifted line as a function of the scattering angle and primary wave-length. If $\Delta\lambda_{1/2}$ is the breadth at say half maximum value then it appeared from this theory that

$$\Delta\lambda_{1/2} = 4\beta_{1/2}\lambda^* \tag{1}$$

[1] DuMond and Kirkpatrick, Phys. Rev. **37**, 136 (1931).

when $\beta_{1/2}$ = the speed of that class of electrons in the scattering body possessing the velocities which cause the broadening at half maximum value and where

$$\lambda^* = \tfrac{1}{2}(\lambda_1^2 + \lambda_c^2 - 2\lambda_1\lambda_c \cos \theta)^{1/2} \tag{2}$$

λ_1 = primary wave-length incident on the scatterer. $\lambda_c = \lambda_1 + (h/mc)(1 - \cos \theta)$ = Compton shifted wave-length for initially stationary electrons. θ = scattering angle.

If the Compton shift is small compared to the wave-length, that is, if λ_1 and λ_c differ by a very small amount relative to either one, then from (1) and (2) it is evident that approximately

$$\Delta\lambda_{1/2} = 4\beta_{1/2}\lambda_1 \sin \tfrac{1}{2}\theta. \tag{3}$$

To a first approximation then the breadth of the Compton line should be proportional to the primary wave-length and to the sine of half the scattering angle if the breadth is to be regarded as an effect of electron velocities.

The paper above referred to reported the results of a test of the dependence of Compton line breadth on scattering angle. Long x-ray exposures on photographic spectrograms of molybdenum K radiation scattered from graphite at three different and very well-defined scattering angles were taken using the multicrystal spectrograph which permits of high spectral resolution and great homogeneity of scattering angle. The breadth of the Compton shifted line was shown by these experiments to obey very satisfactorily the predicted functional dependence on scattering angle. The results constituted only a qualitative test of the dependence of Compton line breadth on primary wave-length however because only two wave-lengths appeared on the spectra, namely $K\beta_1$ and $K\alpha_{12}$ differing by only about ten percent. A slight systematic difference in the breadth of the two corresponding lines did however appear in the right direction and roughly of the right magnitude.

The purpose of the present investigation is to test more thoroughly this dependence of Compton line breadth $\Delta\lambda$ on primary wave-length λ. The point is important because the postulated broadening of the Compton line by the velocities of the scattering electrons can be regarded as a Doppler broadening and the approximate constancy of $\Delta\lambda/\lambda$ is a very characteristic property of such a phenomenon.

The previous paper[1] above referred to contained an analytical solution of the problem of the modified scattering of x-rays by ensembles of moving electrons. This solution can be presented in a very vivid and simple form by introducing two simplifying approximations .Having solved the problem rigorously in the previous paper the authors feel that it is valuable now to present the simple approximate exposition in the interest of clearness.

Simplified Theory of Compton Line Breadth

Two simplifying approximations are made:

1. The change in wave-length due to scattering is a negligible part of the wave-length itself.
2. Relativity is neglected.

In Fig. 1 the x-axis is taken in the bisector of the supplement of the scattering angle θ. The vectors $h\nu_1/c$ and $h\nu_2/c$ representing respectively the momenta of the incident and scattered quanta of radiation are by approximation

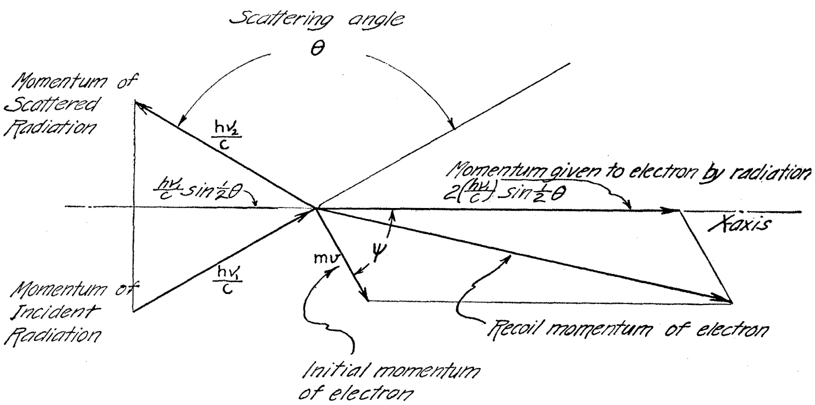

Fig. 1. Diagram of momentum vectors involved in the simplified approximate analysis of Compton scattering by an initially moving electron. The initial electron momentum is here much exaggerated relative to the momentum of the radiation. The plane of the angle ψ need not lie in the plane of the paper. Complete generality is obtained by rotating the plane of the angle ψ around the x-axis. This in no way affects the analysis here presented.

(1) nearly equal. Hence the change in momentum of the radiation, which is the momentum imparted to the electron, is given by

$$2\left(\frac{h\nu_1}{c}\right)\sin\tfrac{1}{2}\theta \tag{4}$$

and is directed along the x-axis.

The recoil momentum of the electron will be the vector sum of this acquired momentum and the initial momentum mv which the electron possesses at the instant of scattering in virtue of its natural motion in the atom or in the crystal lattice. The initial momentum mv may have any direction *in space* making an angle ψ with our x-axis. (In Fig. 1 the planes of the angles ψ and θ are shown both lying in the paper solely for convenience of drawing.)

$$(\text{Recoil momentum})^2 = (mv)^2 + 4\left(\frac{h\nu_1}{c}\right)^2\sin^2\tfrac{1}{2}\theta + 4(mv)\left(\frac{h\nu_1}{c}\right)\sin\tfrac{1}{2}\theta\cos\psi \tag{5}$$

The *recoil energy* is obtained by dividing this equation through by $2m$. Deducting from this result the initial energy $\frac{1}{2}mv^2$ possessed by the electron before scattering we obtain the energy abstracted by the electron from the radiation. Equating this to $h(\nu_1-\nu_2)$ we obtain

$$h(\nu_1-\nu_2) = \frac{2}{m}\left(\frac{h\nu_1}{c}\right)^2\sin^2\tfrac{1}{2}\theta + 2v\left(\frac{h\nu_1}{c}\right)\sin\tfrac{1}{2}\theta\cos\psi. \tag{6}$$

This is converted into wave-lengths by multiplying the last equation by $\lambda/h\nu$ and remembering from approximation (1) that $(\nu_1-\nu_2)/\nu_1=(\lambda_2-\lambda_1)/\lambda_1$. The change in wave-length due to scattering is

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta) + 2\frac{v}{c}\lambda \sin\tfrac{1}{2}\theta \cos\psi. \tag{7}$$

The first term represents the well known Compton shift, while the second term is the modification in the shift caused by the electron's initial velocity. Call this term, l. It may be quite appropriately regarded as a Doppler shift due to the component of the electron's motion along x. Since the electron may move in any direction, $\cos\psi$ can take all values between $+1$ and -1, and l can vary from $-2\beta\lambda \sin\frac{1}{2}\theta$ to $+2\beta\lambda \sin\frac{1}{2}\theta$. Thus the Compton line acquires a breadth

$$4\beta\lambda \sin\tfrac{1}{2}\theta \tag{8}$$

where $\beta = v/c$ for electrons moving in random directions and each having a speed v.

Consider the case of scattering by an ensemble of electrons, all having one and the same speed $\beta = v/c$ and randomly directed velocities. Let us inquire how the intensity will be distributed for such a case between the limits of the line breadth fixed by Eq. (8).

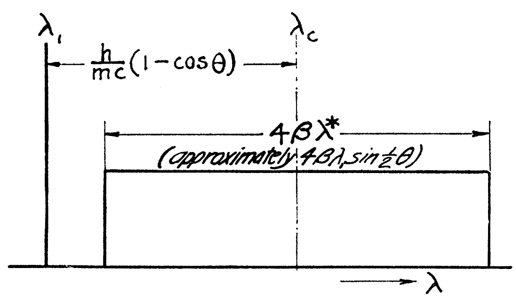

Fig. 2. Spectral energy distribution of modified scattered x-radiation for the ideal case of a monochromatic primary ray scattered by an assembly of electrons all of the same initial speed but with randomly orientated initial directions. The distribution is symmetrically disposed about the wave-length λ_c which is the shifted wave-length for initially stationary electrons. The distribution has a breadth $4\beta\lambda^*$ proportional to the speed of the electrons and is bounded by sharply defined discontinuities on both sides. Between these limits the distribution is uniform.

The chance of scattering by electrons directed so that their angles with the x-axis lie between ψ and $\psi + d\psi$ is:

$$P_\psi d\psi = \tfrac{1}{2}\sin\psi d\psi. \tag{9}$$

The shift away from the center of the Compton line for such electrons is:

$$l = 2\beta\lambda \sin\tfrac{1}{2}\theta \cos\psi \tag{10}$$

$$dl = -2\beta\lambda \sin\tfrac{1}{2}\theta \sin\psi d\psi. \tag{11}$$

Eliminating ψ and $d\psi$ between these equations, we obtain the chance that the shift shall lie between l and $l + dl$. It is:

$$P_l dl = (4\beta\lambda \sin\tfrac{1}{2}\theta)^{-1} dl. \tag{12}$$

The chance of scattering with a given shift l is thus independent of the shift between the limits:

$$-2\beta\lambda \sin \tfrac{1}{2}\theta < l < 2\beta\lambda \sin \tfrac{1}{2}\theta \tag{13}$$

and we conclude that the Compton line from an ensemble of randomly directed electrons, all of speed $\beta = v/c$ is a rectangular spectral distribution, as shown in Fig. 2.

In the real case the Compton line is doubtless due to scattering by electrons having not one speed β alone but a distribution over a considerable range of speeds. The spectral distribution must then be thought of as a superposition of such elementary rectangles of infinitesimal height and of breadth $4\beta\lambda \sin \frac{1}{2}\theta$ arranged one above another in decreasing order of breadth. See Fig. 3.

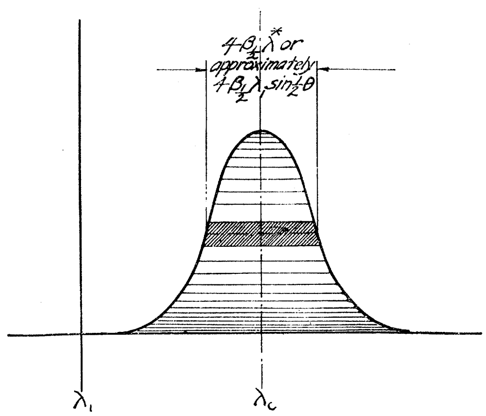

Fig. 3. Spectral energy distribution in Compton line for scattering of initially monochromatic x-radiation by an assembly of electrons distributed over a range of initial speeds and with random initial directions of motion. Each horizontal elementary rectangle is the contribution of the scattered radiation from one particular class of electrons of speed β in the range $d\beta$. The area of each rectangle is proportional to the relative electron population of its class. One of the rectangles will fall midway between the base and summit of the line and the speed corresponding to this class of electrons is designated by $\beta_{\frac{1}{2}}$.

The *area* of each such rectangle is to be made proportional to the electron population of the speed class β to $\beta + d\beta$ which the rectangle represents. Suppose we define the "breadth" of the line arbitrarily for convenience as the breadth at half maximum height and designate it by $\Delta\lambda_{1/2}$. Then

$$\Delta\lambda_{1/2} = 4\beta_{1/2}\lambda \sin \tfrac{1}{2}\theta \tag{14}$$

where $\beta_{1/2}$ is the velocity (referred to light) of the electron class that happens to fall in the line structure at half maximum height. This will remain the same class as we impose variations on θ and λ.[2]

[2] This statement is only true in a limited sense. The velocity $\beta_{\frac{1}{2}}$ will remain constant so long as θ is sufficiently large and λ sufficiently small to insure that practically all of the electrons in the scattering atom shall receive sufficient recoil energy to be set free from the atom in the scattering process. Unless this condition is fulfilled coherent (unmodified) scattering occurs.

We thus have arrived by very simple reasoning at an approximate expression for the behavior of Compton shifted line breadth as a function of scattering angle and primary wave-length. For the purpose of comparison with experimental observation we will use the slightly more accurate results of the

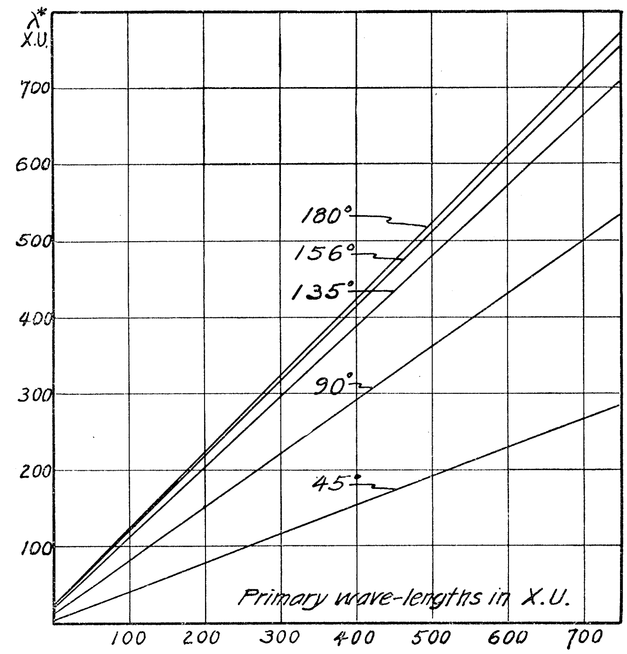

Fig. 4. Graphical representation of the dependence of Compton line breadth on primary wave-length for several scattering angles. The ordinates are values of λ^* which is proportional to the breadth provided $\beta_{\frac{1}{2}}$ remains constant. λ^* is about 35 times as great as the breadth at half maximum for the case of graphite.

exact analysis however. According to Eq. (14) the breadth of the Compton line in wave-length units should be proportional to the primary wave-length. The more exact analysis shows however that the breadth will not tend toward

Unmodified scattering probably occurs as follows: A free electron with specified initial momentum scattering radiation of wave-length λ under a specified angle will receive a definite recoil energy according to the laws of conservation of momentum and energy in the way shown by our theory. A bound electron having the same specified momentum will under the same specified conditions receive this same recoil energy if and only if this recoil energy exceeds the binding energy. If this recoil energy is less than the binding energy the electron will behave as though its mass were the mass of the entire atom and the laws of conservation of momentum and energy will then require an entirely negligible change in wave-length.

If an unmodified line appears this means, according to the above picture, that some of the electrons in the scattering atom part of the time are not contributing to modified scattering and these will on the whole be the more tightly bound electrons which are also the more swiftly moving electrons. There will thus be a slight selection in favor of slower electrons that will tend to reduce $\beta_{\frac{1}{2}}$ slightly when λ is sufficiently large or θ sufficiently small to make the unmodified intensity an appreciable fraction of the modified intensity. For the wave-lengths and scattering angles reported in this paper and the previous paper however we do not believe that such an effect plays any important part in the observed behavior of the shifted line breadth.

zero for very short wave-lengths. This can be seen from an examination of Eqs. (1) and (2). It is evident that as λ_1 tends toward zero λ^* does not vanish but tends toward the value $\frac{1}{2}(h/mc)$ $(1-\cos\theta)$ and the *Compton line for infinitely hard primary radiation* would still have a finite breadth given by

$$\lim_{\lambda\to 0} \Delta\lambda_{1/2} = 2\beta_{1/2}\frac{h}{mc}(1 - \cos\theta). \tag{15}$$

Fig. 4 shows graphically the variation in the breadth of the Compton shifted line as a function of primary wave-length to be expected from the accurate theory. In order to get the ordinates in units independent of the factor $\beta_{1/2}$ which is of no interest in the present investigation, we have plotted in Fig. 4 the course of λ^* as a function of λ_1 for several scattering angles. (λ^* is about 35 times the breadth of the Compton line at half maximum value for the case of graphite.) The graphs of Fig. 4 are not quite straight lines. They may be represented analytically by the equation

$$\lambda^* = \lambda_1 \sin\tfrac{1}{2}\theta\left[1 + \frac{k}{\lambda_1} + \frac{1}{4}\frac{k^2}{\lambda_1^2}\frac{1}{\sin^2\frac{1}{2}\theta}\right]^{1/2} \tag{16}$$

in which

$$k = 2\frac{h}{mc}\sin^2\tfrac{1}{2}\theta.$$

Experimental Test of Dependence of Compton Line Breadth on Primary Wave-length

The multicrystal spectrograph used in this work has been adequately described in previous articles[1,3] and will only be briefly referred to here. We have been delighted to find that in this instrument the fifty separate small Seemann wedge-type crystal spectrographs adjusted to exactly superpose their spectral lines on the negative have stayed in perfectly satisfactory adjustment for a period now exceeding one year. Reasonable care has been taken not to jar the table supporting the instrument. The feet of this table stand on large blocks of especially soft rubber.

Fig. 5 shows schematically the position of the x-ray tube, the graphite scatterer and the multicrystal spectrograph for the case of 156° scattering angle. All the work described in this article was done at or very near to this angle. In this position the tube is situated at a considerable distance from the scatterer, the greatest distance being 120 cm and the least distance 70 cm. This necessitates exposures of the order of one thousand hours but gives in return very fine homogeneity of the scattering angle. Under these conditions the spurious breadth of the Compton line due to inhomogeneity of scattering angle becomes utterly negligible.

The characteristic lines chosen to test the dependence of shifted line-breadth on wave-length were the K lines of molybdenum, silver and tungsten. General Electric water-cooled tubes were used for the molybdenum and

[3] DuMond and Kirkpatrick, Rev. of Scient. Inst. **1**, 88 (1930).

silver exposures, and a water-cooled glass Müller tube was used for the long exposure with tungsten. About half way through the tungsten exposure the Müller tube failed due to a temperature crack in the bulb. It was replaced by another tube exactly like it, great care being taken that the focal spot of the second tube should occupy exactly the same position as the focal spot of the first tube. The tubes ran continuously day and night. The molybdenum tube was held at 20 milliamperes and 50 kilovolts peak, the silver tube at 15 milliamperes and 73 kilovolts peak, and the tungsten tube at 10 milliamperes and 110 kilovolts peak.

An intensifying screen was used behind the film in the case of the exposure with tungsten tube. This is the only case with the multicrystal spectrograph in which we have used an intensifying screen. The screen has not been previously used in order to forestall the objection that our lines might be broadened by poor contact between the intensifying screen and the negative or

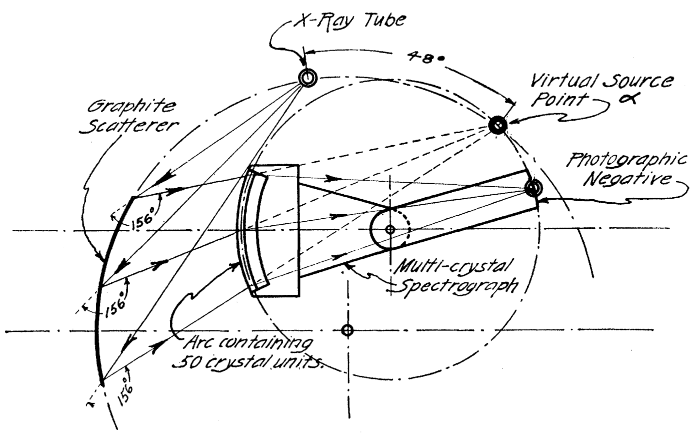

Fig. 5. Schematic diagram showing geometrical dispositions of x-ray tube, scattering body, and multicrystal spectrograph for 156° scattering angle. The setting for molybdenum radiation is the one here shown. For harder primary radiation the virtual source and spectral image are closer together.

that intensities might be falsified by the screen. Of the three exposures here reported the one with tungsten should, according to theory, and does in actual fact, have the narrowest shifted line breadth. It seems quite impossible for anyone to claim that the use of the intensifying screen could have rendered the line sharper or narrower. The sharpness of the primary reference lines appearing on the exposure with tungsten certainly leaves little doubt that the screen can be completely absolved of the suspicion of introducing any spurious effects.

The scatterer in all three cases consisted of a circular arc sawed from a large block of Acheson graphite. The focal spot of the x-ray tube and the arc of the scatterer were accurately located with respect to the point α of the multicrystal spectrograph by means of a plumb bob hanging from a radius arm exactly as described in the previous paper,[1] the focal spot being aligned with this plumb line by sighting from two mutually rectangular directions with auxiliary plumb lines. The scattering angle could be measured with much better than the required accuracy by measuring the angle through which the radius arm must swing in passing from the point α of the multicrystal spectrograph to the focal spot of the x-ray tube. The exact values of the scattering angles for molybdenum, silver and tungsten were respectively 156°, 27′±15′, 155°, 21′±15′ and 154° 15′±1° 30′. The exposure times were respectively 897 hrs., 900 hrs. and 1020 hrs.

The bulk of the radiation from the silver and tungsten x-ray tubes was shielded by placing these tubes each in a housing consisting of a micarta tube wrapped on the outside with a sufficient number of layers of 1/16″ leaded rubber sheeting. The use of leaded rubber which combines the three properties of mechanical flexibility, opacity to x-rays, and good electrical insulation

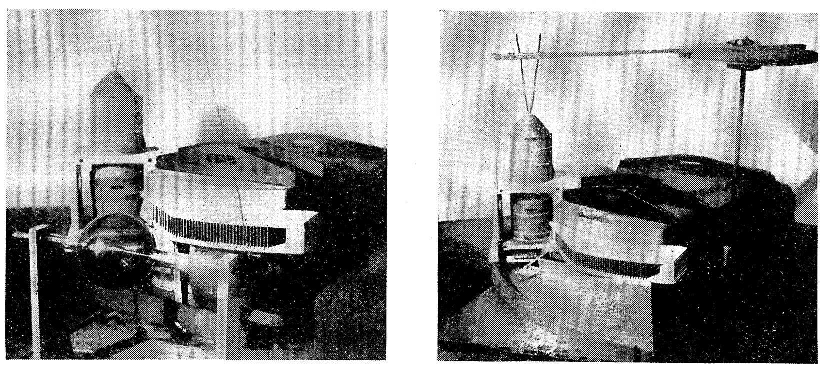

Fig. 6. Showing system of lead baffles used in the case of tungsten radiation. They are accurately located in front of the crystal arc of the multicrystal spectrograph. These baffles beside reducing the fogging due to amorphous scattering serve to shield the negative from strong radiation passing directly under the wedges without suffering reflection from the calcite faces. In the case of tungsten K radiation the reflexion angles are so small that this type of fogging becomes important. The right photograph shows the graphite scatterer in place for the long exposure with scattered radiation. The left photograph shows the scatterer removed and a tungsten x-ray tube situated on the swinging wooden sector for the short exposure to give the reference lines on the edge of the negative.

makes possible a much smaller, more compact and convenient tube housing than could be obtained with metallic lead, and eliminates the necessity for a bushing insulator. A hole was cut through the leaded rubber and micarta tube of just the right size and shape to permit an x-ray beam to illuminate the entire graphite scatterer. Another small hole was also cut at a point 90° around the cylinder from the above window to facilitate sighting the target with the plumb line.

The exposure with tungsten radiation required a considerable amount of preliminary work. The wedges in the crystal holders of the multicrystal spectrograph are of brass and have an angle of 120°. Tests were made, using direct radiation, with a sample unit to determine whether these brass wedges would be sufficiently opaque to the tungsten K radiation to give sharp, well-defined lines. They were found to be satisfactory.

The grazing angle with calcite for the tungsten $K\alpha$ lines is only about two degrees. The radiation which passes directly through under the wedge without reflection from the calcite has therefore a direction which differs from that of the $K\alpha$ lines by only two degrees. A system of fifty baffles of sheet lead 1/8″ thick was therefore designed and built with great care so that each baffle would cut off the straight-through radiation to one of the crystal units. The radiation coming from the scatterer in the proper direction to form the spectrum over the region of interest could however pass between the baffles. The alignment of the baffles was carefully checked by shining light from an incandescent bulb placed behind a paper slot situated on the film holder in the region of interest, the light passing backward to the crystals along the paths later to be followed by the x-rays (in reverse sense). The straight-through beams and the reflected beams from the crystals could be easily observed and the baffle holder could thus be adjusted so that the former were cut off and the latter permitted to pass. In Fig. 6 the lead baffles can be plainly seen.

It is a well-known fact that for a given scattering material and scattering angle the ratio of unmodified line intensity to modified line intensity diminishes with diminishing wave-length. For tungsten radiation scattered at large angles from graphite we did not expect to obtain any unmodified line at all and as a matter fact none appeared. Ordinarily the unmodified lines serve admirably the purpose of fiducial lines to establish the wave-length scale. In the case of tungsten the expected absence of the unmodified lines made it advisable to establish reference lines on the film. This was done by making an auxiliary exposure with primary tungsten radiation from a Coolidge tube placed in front of the spectrograph approximately in the position occupied by the scatterer which had been removed for the purpose. This exposure was made just after the 1020 hr. exposure with scattered radiation. A lead shield was introduced in front of the negative during this exposure with primary radiation so that only the edges of the negative were exposed. This was done to avoid all risk of fogging or falsifying the valuable 1020 hr. exposure. Of course great care was taken not to disturb the negative between the two exposures. The Coolidge tube used for this reference line exposure was gradually shifted by means of a screw so that it passed in front of a large number of crystals in the multicrystal spectrograph, thus giving a fair integration of the average position of the K lines from the crystals, rather than the position of these lines from one single crystal which might be very slightly displaced from the mean. In Fig. 6 the tube used for the reference line exposure can be seen standing on the swinging wooden sector which permitted moving it in front of the crystals.

Fig. 7 is a reproduction of the three negatives of molybdenum, silver and tungsten K radiation scattered from graphite. The decrease in Compton line breadth with decreasing primary wave-elngth is very clearly evident from an inspection of these reproductions. Attention is called to the sharpness of the unshifted lines and in the case of the tungsten exposure to the sharpness of the reference lines. These bear witness as to the good state of adjustment of the multicrystal spectrograph and preclude the possibility of the objection being raised that the diffuseness of the shifted line might be caused by some instrumental defect. The narrowness of the Compton line in the case of the tungsten exposure is convincing evidence that the multicrystal spectrograph

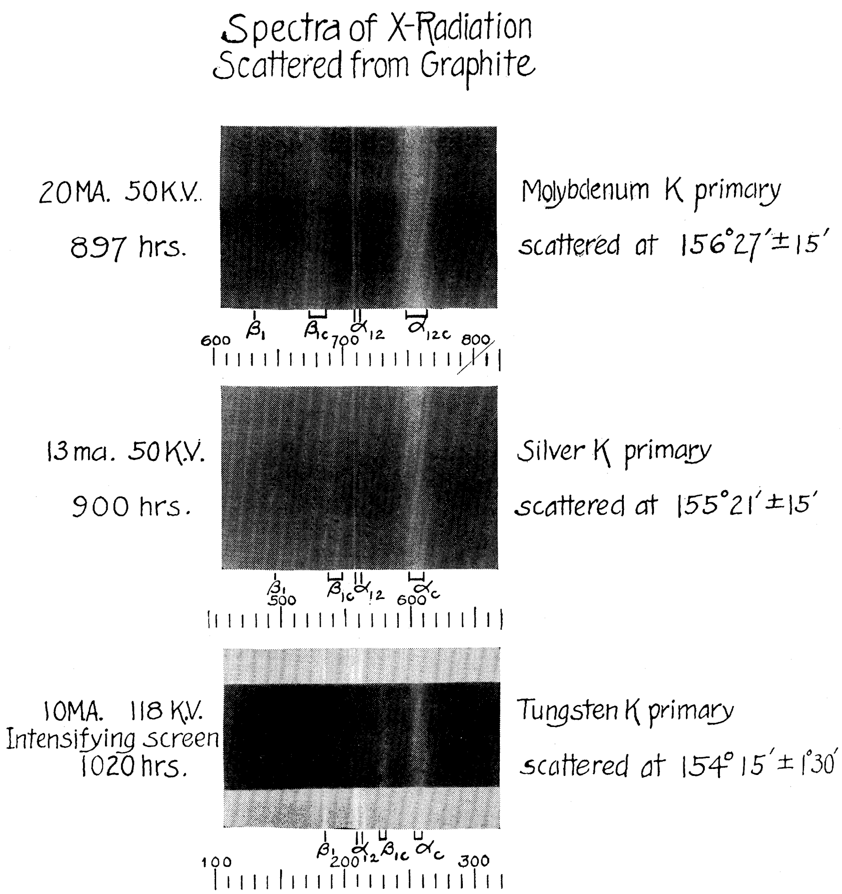

Fig. 7. Reproduction of three spectra of K radiation scattered by graphite at about 156°. The dispersion in wave-length units is the same on all three exposures as can be seen from the fact that the Compton shift for the $K\alpha$ lines is the same in all three cases. (At such a large scattering angle the slight variation in scattering angle between the three cases produces no detectable difference in shift.) Note the very evident diminution in the breadth of the shifted line measured in wave-length units as the primary wave-length diminishes. This is not an effect of over or under exposure or halation as the maximum density of blackening in the Compton line is very nearly the same in all three cases. A very considerable over exposure only very slightly broadens the narrow primary lines as can be seen in the case of the reference lines of tungsten. An intensifying screen was used in the case of tungsten only. These reproductions have been slightly intensified by the use of contrast printing paper to compensate for the loss in the half-tone process. A scale of wave-lengths in x-units accompanies each spectrum.

can give a narrow shifted line when such a narrow line is present. It satisfactorily eliminates the possible objection discussed in the previous paper[1] that our shifted line breadths might be exaggerated by multiple scattering or by air scattering because there is no reason to expect such spurious scattering effects to be more pronounced for molybdenum or silver K radiation than

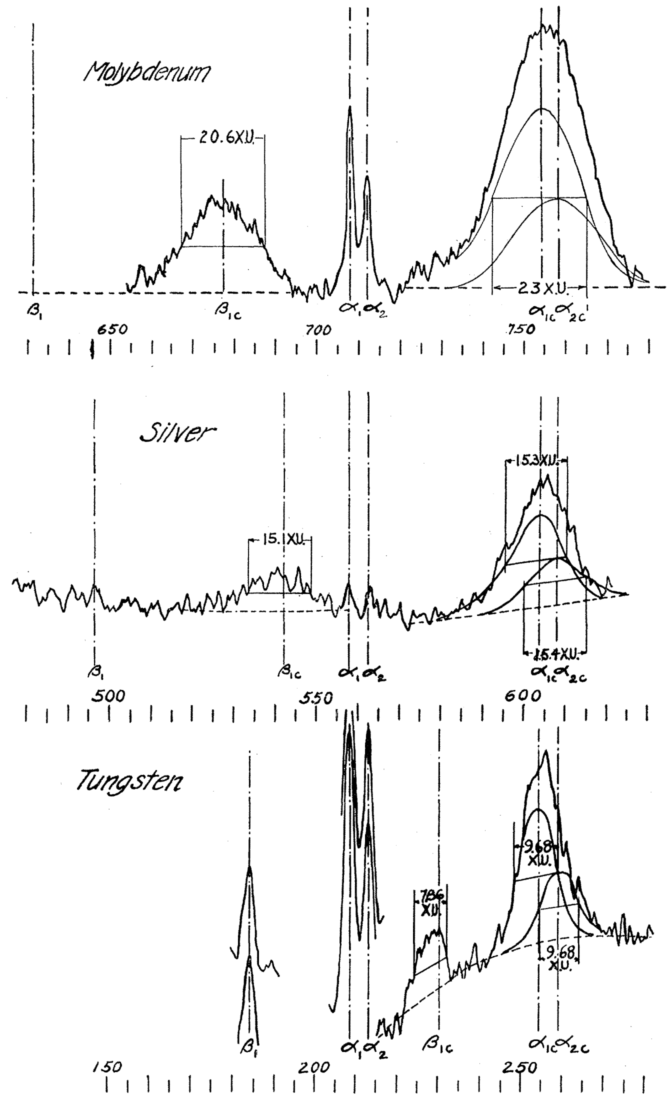

Fig. 8. Three typical microphotometer curves taken from the negatives shown in Fig. 7. A number of such curves were run on each negative in different regions of the height of the spectral lines. In the case of the tungsten spectrum the position of the reference lines had to be recorded by making two separate runs across the two edges of the film. This could be easily accomplished as the table carrying the negative on the microphotometer can be shifted laterally accurately normal to its direction of travel.

for tungsten K radiation. In fact, the evidence from the photographs shown in Fig. 7 seems to us to offer inescapable proof of the reality of the predicted and observed behavior of Compton line breadth.

In Fig. 8 typical microphotometer curves made from the above mentioned three negatives are shown. The shifted $K\alpha$ line is of course a superposition of two shifted lines corresponding to the two components of the primary $K\alpha$ doublet. The observed shifted $K\alpha$ line was graphically decomposed into its two components by the method already described in the previous paper.[1] This decomposition is a quite determinate process because of the fortunate fact that the relative intensity and wave-length separation of the two otherwise identical components is accurately known. In Fig. 8 the decomposed components are shown.

TABLE I. *Breadths in x-units* $\Delta\lambda_{\frac{1}{2}}$ *of shifted lines.*

Tungsten K rad.				Silver K rad.				Molybdenum K rad.		
Run	α_{1c}	α_{2c}	β_{1c}	Run	α_{1c}	α_{2c}	β_{1c}	Run	α_{1c}	β_{1c}
1	7.85	7.85	8.46	1	15.3	15.4	15.1	1	22.8	22.4
2	10.6	10.3	9.65	2	15.7	16.5	17.5	2	21.2	19.4
3	10.55	10.28	8.16	3	15.4	14.8	17.2	3	23.0	20.6
4	9.68	10.00	7.86	4	Bad	Blemish	16.0	4	23.0	20.6
5	9.68	9.68	7.86	5	15.9	14.8	15.1	5	21.9	21.5
				6	16.6	16.6	16.6			
				7	17.22	17.85	17.85			
Av.	9.67	9.62	8.39	Av.	16.02	15.99	16.48	Av.	22.4	20.9
	9.65		8.39		16.0		16.48		22.4	20.9

In Table I the breadths at half maximum of all the shifted lines are listed for all the microphotometer curves together with average breadths for the various wave-lengths. In Fig. 9 the individual and average breadths have been plotted as a function of primary wave-length for comparison with the theoretically predicted curve. Since the point to be tested is the functional dependence of breadth on primary wave-length and not the absolute value of the breadth one of the six observed breadths, namely the breadth of shifted Mo$K\alpha_1$ was arbitrarily fitted to the theoretically predicted curve, thus establishing the scale to which the other five observed breadths were plotted. The breadth variation checks the theoretical prediction to a precision well within the range to be expected.[4]

SIGNIFICANCE OF RESULTS

The two new physical effects whose prediction and observation we have reported in this paper and the previous one deserve a certain amount of critical discussion.

The quantitative fulfillment of the theoretically predicted behavior of Compton shifted line breadth is very clear and distinct. The simple and al-

[4] In Fig. 9 it will be noted that the observed breadths for the shifted tungsten lines fall slightly above the theoretical curve. If these were adjusted to fit the curve then the breadths for silver and molybdenum shifted radiation would fall below the curve. There is a bare possibility that this may in part at least be caused by the above mentioned decrease in $\beta_{\frac{1}{2}}$ by a selection in favor of slower electrons for the cases of softer radiation where unmodified scattering occurs. (See footnote 3) The relative faintness of the unmodified lines makes it appear much more likely however that the slight deviation (only about 2 X.U.) in the points for tungsten is merely an experimental error.

most classical nature of the theory so satisfactorily fulfilled must not however be interpreted as an indication that our results are in conflict with wave-mechanics. There is a tendency at present for physicists to feel suspicious of any theory not formulated in the language of the wave equation or of matrices, and a positive distrust is aroused if a theory becomes easy to visualize. The assumptions on which our theory of Compton shifted line breadth rests

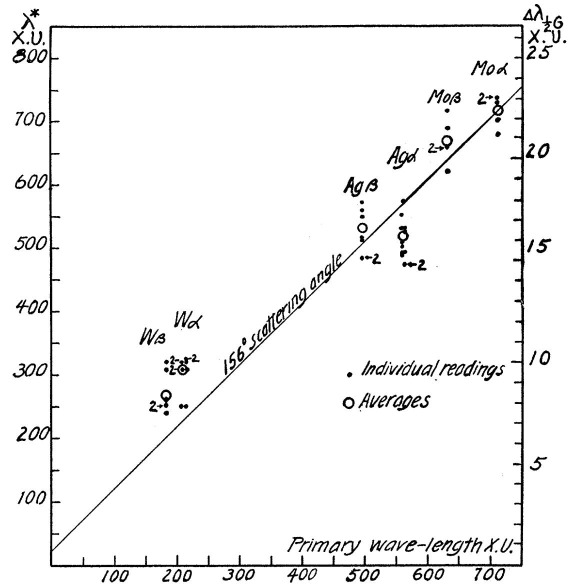

Fig. 9. Comparison of observed breadths of Compton lines for different primary wave-lengths with the theoretically predicted functional dependance of breadth on primary wave-length. The ordinate scale on the left is λ^* in X.U. The ordinate scale on the right is the breadth in x-units of the Compton line at half maximum value adjusted to fit the theoretical curve at one point only, namely the observed breadth of shifted Mo $K\alpha_{12}$.

are in complete accord with wave mechanics and do not in any way violate the Heisenberg principle of uncertainty. They are:

(1) and (2) Conservation of energy and momentum in individual processes.

(3) Probability of scattering by a given class of electrons (of speeds β to $\beta+d\beta$) proportional to the population of that class.

(4) Electron binding energy negligible compared to energy transferred to electron in scattering process.

(5) Initial electron velocity small compared to velocity of light.

It is clear that the uncertainty principle is not violated by our theory. The electron's *position* in our experiment is wholly undetermined and unspecified. Its velocity is therefore theoretically determinable with unlimited precision. This electron velocity can in fact be thought of as having been measured by observing the Doppler change in wave-length which it causes in the radiation scattered by the electron. Heisenberg discusses this very method of measuring the velocity or momentum of an electron in his book "The Physical Principles of the Quantum Theory" (page 25). *The broadening of the Compton line may therefore quite legitimately be thought of as a Doppler broadening caused by the initial velocities of all the electrons in the scattering body responsible for x-ray scattering.* (In the case of tungsten radiation scattered at large angles from graphite there seems to be little doubt that this means all six of the electrons in the carbon atom.)

An analysis of the structure of the broadened Compton line furnishes information as to the probability distribution of electron momenta in atoms. Indeed the *momentum eigenfunktion* of any atom can now be investigated by our method as readily as the *position eigenfunktions* have been determined in diffraction experiments. Analyses of Compton line structures into momentum eigenfunktions are soon to be published. Preliminary work of this kind indicates that the line structures so far obtained yield momentum eigenfunktions in good accord with the predictions of modern quantum mechanics.

It seems fair therefore to say that the broadening of the Compton line and the behavior of its breadth as a function of scattering angle and primary wavelength furnishes us with a direct experimental proof of the dynamic nature of the structure of atoms and solid bodies. The astounding electron velocities postulated by Bohr in his atom model which are still present with but slight modification in the quantum mechanics under the more sophisticated name of "momentum eigenfunktions" may be said at last to have been experimentally observed in about as real a sense as anything else in Physics can be observed.

The results of this work are so clearly defined that the reality of the much narrower Compton lines obtained recently by several investigators seems in our opinion to be very doubtful. This raises an interesting point in connection with determinations of the constant h/mc by measurements of Compton shift. It is evident that the smallest percentage error in the measured values will occur when the ratio of Compton line breadth to Compton shift is a minimum. The laws expressed in Eqs. (1), (2) and (3) show that the best conditions for measurement of h/mc occur at very short primary wave-lengths and large scattering angles. These conditions are fairly well satisfied by our exposure with tungsten radiations scattered by graphite at 156°. Precise measurements of shift from this exposure and others will be made the subject of a paper to be published in the near future. In our opinion the breadth of the Compton line even for these optimum conditions is too great to warrant an accuracy much better than one half percent in the determinations of h/mc and we fear that higher accuracy than this claimed by other investigators on the basis of apparently very narrow Compton lines has been illusory.

Further work with the multicrystal spectrograph will consist of an exploration of Compton line breadths and structures for various scattering materials.

In conclusion we wish to express our deep appreciation of the facilities put at our disposal by the Norman Bridge Laboratory and of the interest and encouragement given us by Dr. R. A. Millikan. This investigation which has involved a great deal of expense has been carried out with the aid of the Seeley W. Mudd X-Ray Research Fund. We take this opportunity to express our gratitude for this financial aid.

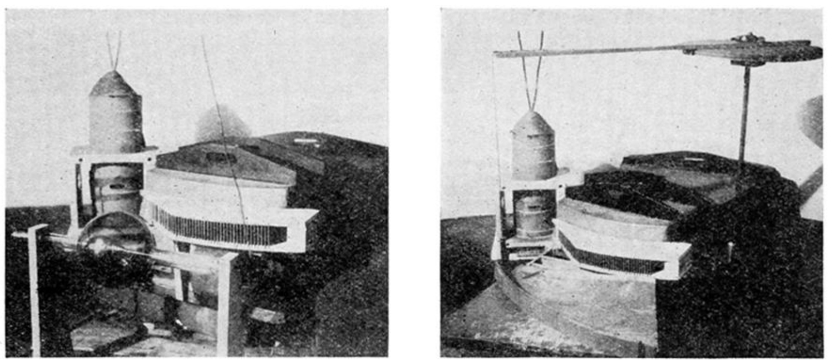

Fig. 6. Showing system of lead baffles used in the case of tungsten radiation. They are accurately located in front of the crystal arc of the multicrystal spectrograph. These baffles beside reducing the fogging due to amorphous scattering serve to shield the negative from strong radiation passing directly under the wedges without suffering reflection from the calcite faces. In the case of tungsten K radiation the reflexion angles are so small that this type of fogging becomes important. The right photograph shows the graphite scatterer in place for the long exposure with scattered radiation. The left photograph shows the scatterer removed and a tungsten x-ray tube situated on the swinging wooden sector for the short exposure to give the reference lines on the edge of the negative.

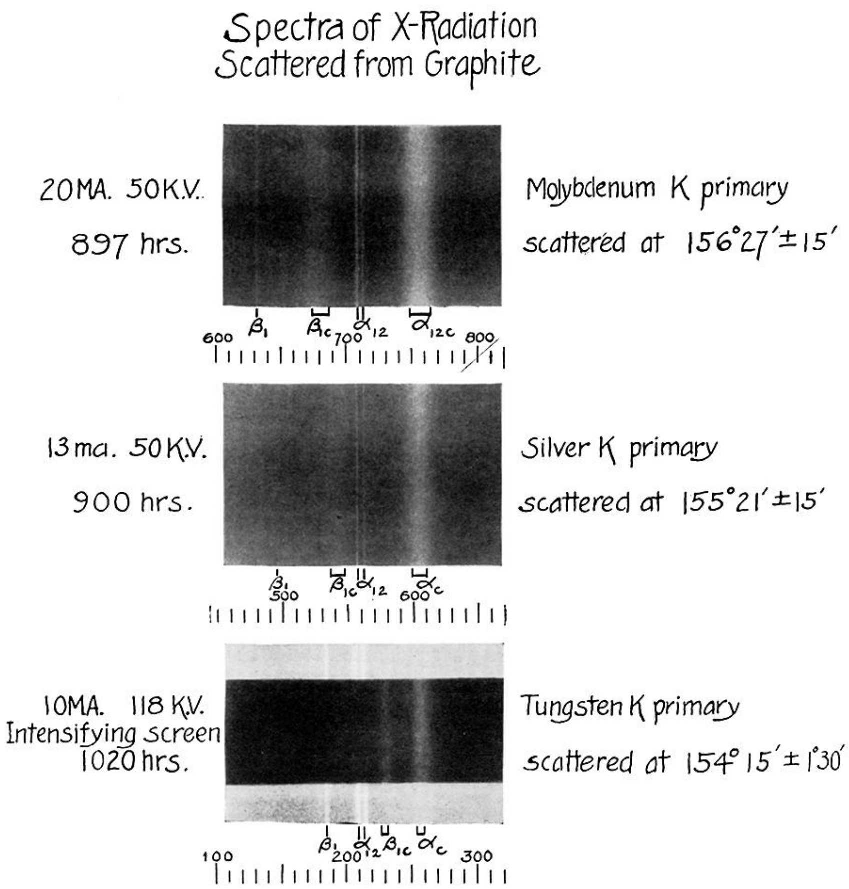

Fig. 7. Reproduction of three spectra of K radiation scattered by graphite at about 156°. The dispersion in wave-length units is the same on all three exposures as can be seen from the fact that the Compton shift for the $K\alpha$ lines is the same in all three cases. (At such a large scattering angle the slight variation in scattering angle between the three cases produces no detectable difference in shift.) Note the very evident diminution in the breadth of the shifted line measured in wave-length units as the primary wave-length diminishes. This is not an effect of over or under exposure or halation as the maximum density of blackening in the Compton line is very nearly the same in all three cases. A very considerable over exposure only very slightly broadens the narrow primary lines as can be seen in the case of the reference lines of tungsten. An intensifying screen was used in the case of tungsten only. These reproductions have been slightly intensified by the use of contrast printing paper to compensate for the loss in the half-tone process. A scale of wave-lengths in x-units accompanies each spectrum.